Single-Particle Spin-Orbit Strengths of the Nucleon and Hyperons by SU_6 Quark-Model

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Abstract

The quark-model hyperon-nucleon interaction suggests an important antisymmetric spin-orbit component. It is generated from a color analogue of the Fermi-Breit interaction dominating in the one-gluon exchange process between quarks. We discuss the strength S_B of the single-particle spin-orbit potential, following the Scheerbaum's prescription. Using the SU_6 quark-model baryon-baryon interaction which was recently developed by the Kyoto-Niigata group, we calculate NN, ΛN and ΣN G-matrices in symmetric nuclear matter and apply them to estimate the strength S_B. The ratio of S_B to the nucleon strength $S_N \sim -40$ MeV$\cdot$fm^5 is $S_\Lambda/S_N \sim 1/5$ and $S_\Sigma/S_N \sim 1/2$ in the Born approximation. The G-matrix calculation of the model FSS modifies S_Λ to $S_\Lambda/S_N \sim 1/12$. For S_N and S_Σ, the effect of the short-range correlation is comparatively weak against meson-exchange potentials with a short-range repulsive core. The significant reduction of the Λ single-particle potential arises from the combined effect of the antisymmetric LS force, the flavor-symmetry breaking originating from the strange to up-down quark-mass difference, as well as the effect of the short-range correlation. The density dependence of S_B is also examined.

Key words: YN interaction, quark model, G-matrix, hyperon single-particle potential, spin-orbit interaction
PACS: 13.75.Cs,12.39.Jh,13.75.Ev,24.85.+p

1 Introduction

Though the quantum chromodynamics (QCD) is believed to be the fundamental theory of the strong interaction, it is still too difficult to apply the QCD directly to two-baryon systems. At this stage a number of effective models have been proposed to understand the nucleon-nucleon (NN) and hyperon-nucleon

(YN) interactions from basic elements of quarks and gluons [1]. Among them the non-relativistic quark model has a unique feature that enables us to take full account of the dynamical motion of the two composite baryons within a framework of the resonating-group method (RGM) [2]. The model describes confinement with a phenomenological potential and uses the quark-quark (qq) residual interaction consisting of a color analogue of the Fermi-Breit (FB) interaction. In the last several years, it was found that such a naive model does not produce medium- and long-range interactions, but can give a realistic description of the NN and YN interactions if meson-exchange effects are properly taken into account in the model.

A simultaneous description of the NN and YN interactions has recently been achieved by two groups. One is the SU_6 quark model, RGM-F [3,4], FSS [5–7] and RGM-H [6,7], by the Kyoto-Niigata group, [1] and the other is the SU_3-chiral symmetry quark model [8–10] by the Beijing-Tübingen group. In these models, the spin-flavor SU_6 or chiral-symmetric effective meson-exchange potentials (EMEP) generated from scalar and pseudo-scalar meson exchanges between quarks are incorporated. It was found that the flavor-nonet scalar mesons play an important role in describing the NN and YN interactions in a single framework with a unique set of model parameters. We stress that a simultaneous and realistic description of the NN and YN interactions is very important, since the experimental data for the YN interaction are at present very limited, and thus one has to rely on the theoretical consistency of the framework in order to make best use of the rich experimental information on the NN interaction.

One of the features of the quark-model description for the NN and YN interactions is that the antisymmetric LS force ($LS^{(-)}$ force) originating from the FB spin-orbit interaction is considerably strong in the strangeness $S = -1$ and the isospin $I = 1/2$ channel [11–13]. Since the signs of the ordinary LS force and the $LS^{(-)}$ force are opposite in the ΛN interaction, this strong $LS^{(-)}$ force is vital to produce very small spin-orbit (ℓs) splitting for the Λ single-particle (s.p.) states. This is consistent with the early experimental observation that the s.p. spin-orbit term in $40 \geq A \geq 12$ nuclei is almost zero from the analysis of the recoilless (K^-, π^-) reaction. [14] More recently, preliminary results of the γ-ray spectroscopy for ${}^{9}_{\Lambda}$Be and ${}^{13}_{\Lambda}$C hypernuclei seem to indicate very small ℓs splitting in these nuclei. [15] In view of the recent progress of experimental techniques, a quantitative analysis of s.p. ℓs

[1] Difference of these three models lies only in how to deal with the spin-flavor (-color) factors of the quark-exchange kernel in EMEP. In FSS and most of RGM-H these factors are exactly calculated, while in RGM-F they are approximated to be proportional to those of the exchange normalization kernel. RGM-H uses the latter approximation partly: i.e., solely for the isoscalar-type scalar-meson (ϵ and S^*) exchanges.

potentials appears important. The purpose of this paper is to extend the Scheerbaum's formulation [16] for the nucleon s.p. ℓs potentials to hyperons interacting with nucleons via the non-local interaction, and to examine in detail the s.p. ℓs strengths of N, Λ and Σ, first in the Born approximation of the quark-exchange kernel, and then in the G-matrix calculation for our realistic quark-model NN and YN interactions.

Since the spin-orbit interaction between baryons is essentially short-ranged, a number of authors have payed attention to the FB LS force, trying to understand its microscopic origin from the quark degree of freedom. Here we briefly review some typical investigations, in which the spin-orbit forces of the NN and YN interactions are treated in the $(3q)$-$(3q)$ RGM. In the WKB-RGM localization techniques of the quark-exchange kernel, Suzuki and Hecht [11] calculated LS potentials, originating from the symmetric (sLS) and antisymmetric (aLS) pieces of the FB interaction.[2] They assumed the same strange and up-down quark masses and neglected the flavor symmetry breaking (FSB). This restriction was removed in [12]. After the correction of the sign error of the original paper, they found that the sLS and aLS spin-orbit terms have same sign and therefore reinforce each other, giving rise to an attractive spin-orbit potential in the ^3O state and a repulsive potential in the ^3E state for the NN interaction.[3] Morimatsu *et al.* [13] used only the sLS piece, but took into account the effect of FSB in a simple approximation. In these studies, a main interest is naturally the $LS^{(-)}$ force which involves the simultaneous spin-flip and the flavor exchange of the hyperon and the nucleon, a typical feature of the non-identical baryon systems. The potential concept used in [13] is not based on the RGM kernel, but on the energy surface of the so-called generator-coordinate method (GCM) kernel. Using the folding procedure for the GCM kernel, they calculated, for the first time, the quark-model predictions for the s.p. ℓs potentials of the nucleon and hyperons in symmetric nuclear matter. Although their absolute values of the s.p. ℓs strengths are somewhat too large, they obtained the relative ratio, $U_N : U_\Lambda : U_\Sigma = 1 : 0.21 : 0.55$, which is very close to our prediction $1 : 1/5 : 1/2$ in the Born approximation given in this paper. On the other hand, He, Wang and Wong [18] compared the quark-model potentials with the Paris (for NN) and the Nijmegen potentials in the form of the Born amplitudes. They explicitly introduced a core radius c, in order to take into account the effect of the short-range correlation in the Nijmegen hard-core model D [19] and model F [20]. This procedure was also adopted by the Jülich group to show the relative strength of the LS and $LS^{(-)}$ forces in their one-boson exchange potential (OBEP) model [21]. Through all of these studies, it is now generally recognized that the quark-exchange kernel from the FB interaction leads to the spin and flavor dependence which is qualitatively very similar to that of OBEP, thus

[2] Here we follow the notation given in Eq. (5.2) of [23].
[3] Ref. [17] cites [11] erroneously at this point.

yielding a possible alternative to the meson-exchange description of the LS interaction by vector and scalar mesons. [4]

Strictly speaking, the LS force cannot be discussed independently of other pieces of interaction. Apparently the LS force is influenced by the description of the short-range correlation which is different between the meson-exchange model and the quark model. The LS force also depends on how to derive the s.p. ℓs potentials in the finite nuclei from the original NN and YN interactions in the free space. In fact, the first issue is the major motivation for any realistic quark models for the NN and YN interactions. For example, Yang *et al.* [10] discussed the difference of the one-gluon exchange (OGE) process and the scalar-meson nonet exchange (OSE) introduced between quarks in the framework of the chiral SU_3 quark model. Since their LS force is too weak in the NN sector because of several reasons, they reinforced the sLS term of OGE by a factor of 3.1 and that of OSE by a factor of 4.8. Through this prescription, they argued that a sizable OGE component, which would definitely result in a quite strong $LS^{(-)}$ force in the $I = 1/2$ channel, is not favorable, since it leads to an unphysical resonance in the ΛN channel. This is more or less a correct statement as long as the LS components of the FB interaction is concerned. However, our result in the model RGM-H [6,7] implies that there exists a solution which reproduces the necessary LS force in the NN interaction without introducing any enhancement factor, and still reproduces the observed Λp, $\Sigma^+ p$ and $\Sigma^- p$ differential cross sections reasonably well. The main difference between the two models lies in the choice of the harmonic oscillator constant b of the $(3q)$ clusters and the magnitude of the quark-gluon coupling constant α_S. [5] The Beijing, Tübingen and Salamanca groups use $b \sim 0.5$ fm^{-1} and $\alpha_S \sim 0.5$, while RGM-F, FSS and RGM-H use $b \sim 0.6$ fm^{-1} and $\alpha_S \sim 2$. Since the LS force is short-ranged, it is very sensitive to the magnitude of the size parameter b. It is sometimes claimed that our α_S is too big, compared with the QCD coupling constant, and is contradictory to the experimental fact that there seems no spin-orbit splitting existing in the negative-parity excited states of baryons (especially, the nucleon and Δ). We should, however, keep in mind that our α_S is merely a model parameter in the nonperturbed region, which has very little to do with the real QCD coupling constant. The explicit value is determined from the best fit to the experimental

[4] There exists, however, appreciable quantitative difference between predictions by various versions of OBEP and the quark model. For example, the Nijmegen potentials generally predict a rather small $LS^{(-)}$ force, compared with that of the quark model.

[5] The model RGM-H (nor the other versions, RGM-F and FSS) does not include the LS component from the scalar-meson exchange, while it is included in the chiral SU_3 quark model. However, the incorporation of even more sophisticated EMEP involving vector mesons does not change this situation. A new version of our quark model in this direction will be published elsewhere.

data in the present framework. The second point is the so-called "missing LS force" problem in the P-wave baryons. Fujiwara [22] has shown that the seemingly small spin-orbit splitting of the P-wave baryon spectrum can be explained by the dispersive effect due to the resonance nature of these P-wave baryons embedded in the baryon-meson continua. In other words, the missing LS force problem of the P-wave baryons does not necessarily indicate that the FB interaction is inappropriate as a residual interaction of the non-relativistic quark model.

In this paper, we apply the Scheerbaum's discussion [16] for the strength of the nucleon s.p. ℓs potential to the formulation of the quark-model invariant amplitudes developed in [23]. The strength factor S_B for the hyperon s.p. ℓs potential in the Thomas form is explicitly derived. Two different kinds of approaches are attempted in the Born approximation. One is to use the Wigner transform at $\boldsymbol{p} = 0$ in the WKB-RGM formalism as an effective local potential in low-energy processes, and the other is the P-wave approximation for the dominant contribution to the LS invariant amplitudes. Both methods involve some kind of averaging procedure for the spatial integrals and leave one momentum as an input parameter. This momentum dependence, however, is generally very weak. One can thus adopt the zero-momentum limit, in which these two methods give the same result, yielding very simple expressions for S_B. We consider spin-saturated (s.s.) finite nuclei or symmetric nuclear matter. The most reliable description of S_B is therefore formulated through the nuclear-matter approximation of the G-matrix invariant amplitudes.

We present in Section 2 basic formulae to calculate S_B. After introducing two kinds of approximations to the spatial integrals for the LS Born amplitudes in Subsection 2.2, a method of G-matrix calculation is discussed in Subsection 2.3. In Section 3 we give analytic expressions of S_B in the simplest Born approximation, and use them to examine the characteristic structure of the s.p. ℓs potentials. The G-matrices calculated in nuclear matter are used to obtain a more realistic estimate for S_B. The strength S_Λ turns out to be very small because of the cancellation between LS and $LS^{(-)}$ components. The short-range correlation is found to further reduce S_Λ to be less than $(1/10)S_N$. Section 4 is devoted to a summary.

2 Formulation

2.1 Strengths of hyperon single-particle spin-orbit potentials

We start from the RGM equation for the $(3q)$-$(3q)$ system [3,6]:

$$\left[\varepsilon_\alpha + \frac{\hbar^2}{2\mu_\alpha} \left(\frac{\partial}{\partial \boldsymbol{R}} \right)^2 \right] \chi_\alpha(\boldsymbol{R}) = \sum_{\alpha'} \int d\boldsymbol{R}' \; G_{\alpha\alpha'}(\boldsymbol{R}, \boldsymbol{R}'; E) \; \chi_{\alpha'}(\boldsymbol{R}') \quad , \quad (1)$$

where the $G_{\alpha\alpha'}(\boldsymbol{R}, \boldsymbol{R}'; E)$ is composed of various pieces of the interaction kernels as well as the direct potentials of EMEP. The subscript α stands for a set of quantum numbers of the channel wave function; $\alpha = [1/2(11)\, a_1, 1/2(11) a_2]$ $SS_z YII_z; \mathcal{P}$, where $1/2(11)a$ is the spin and SU_3 quantum number in the Elliott notation $(\lambda\mu)$, $a = YI$ the flavor label of the octet baryons ($N = 1(1/2)$, $\Lambda = 00$, $\Sigma = 01$ and $\Xi = -1(1/2)$), and $\mathcal{P}$ is the flavor-exchange phase. In the NN system with $a_1 a_2 = NN$, $\mathcal{P}$ is actually redundant since $\mathcal{P} = (-1)^{1-I}$. The relative energy ε_α in the channel α is related to the total energy E of the system through $\varepsilon_\alpha = E - E^{int}_{a_1} - E^{int}_{a_2}$. According to [23], we introduce the basic Born kernel of Eq. (1) through

$$
\begin{aligned}
M_{\alpha\alpha'}(\boldsymbol{q}_f, \boldsymbol{q}_i; E) &= \langle\, e^{i\boldsymbol{q}_f \cdot \boldsymbol{R}} \,|\, G_{\alpha\alpha'}(\boldsymbol{R}, \boldsymbol{R}'; E) \,|\, e^{i\boldsymbol{q}_i \cdot \boldsymbol{R}'} \rangle \\
&= \langle\, e^{i\boldsymbol{q}_f \cdot \boldsymbol{R}} \eta^{SF}_\alpha \,|\, G(\boldsymbol{R}, \boldsymbol{R}'; E) \,|\, e^{i\boldsymbol{q}_i \cdot \boldsymbol{R}'} \eta^{SF}_{\alpha'} \rangle \quad ,
\end{aligned}
\tag{2}
$$

where η^{SF}_α is the spin-flavor wave function at the baryon level, defined in Eq. (2.9) of [23].

In the following we restrict ourselves to the spin-saturated (s.s.) nuclei and apply the Scheerbaum's prescription for the s.p. spin-orbit strengths, first to the quark-exchange kernel $G_{\alpha\alpha'}(\boldsymbol{R}, \boldsymbol{R}'; E)$, secondly to the G matrices obtained by solving the corresponding Bethe-Goldstone equation [25]. We call the first prescription the Born approximation, and the second one a realistic calculation. Suppose G is the quark-exchange kernel $G(\boldsymbol{R}, \boldsymbol{R}'; E)$ or the G-matrix. We calculate s.p. energy

$$
E^{s.s.}_v = \sum_c \ \langle\, vc \,|\, G \,|\, vc - cv \,\rangle \quad ,
\tag{3}
$$

for the spin-orbit interaction. The two-particle interaction G is assumed to be expressed as

$$
\begin{aligned}
\langle\, \boldsymbol{k}_1 \boldsymbol{k}_2 \,|\, G \,|\, \boldsymbol{k}'_1 \boldsymbol{k}'_2 \,\rangle &= \delta(\boldsymbol{K}_{12} - \boldsymbol{K}'_{12}) \ \langle\, \boldsymbol{k}_{12} \,|\, G \,|\, \boldsymbol{k}'_{12} \,\rangle \\
&= \delta(\boldsymbol{K}_{12} - \boldsymbol{K}'_{12}) \ \frac{1}{(2\pi)^3} \ M(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12}) \quad ,
\end{aligned}
\tag{4}
$$

where $\boldsymbol{K}_{12} = \boldsymbol{k}_1 + \boldsymbol{k}_2$ and $\boldsymbol{k}_{12} = (\xi \boldsymbol{k}_1 - \boldsymbol{k}_2)/(1+\xi)$ with $\xi = (M_2/M_1)$ are the center-of-mass and relative momenta, respectively. In the case of the G-matrix, $M(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12})$ may depend on $(\boldsymbol{K}_{12})^2$ and the starting energy as well. It is convenient to use the invariant kernel $M^\Omega(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12})$, by which the Born kernel Eq. (2) is expressed as

$$
M(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12}) = \sum_\Omega M^\Omega(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12}) \ \mathcal{O}^\Omega(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12}) \quad .
\tag{5}
$$

Here we only consider $\Omega = LS,\ LS^{(-)}$ and $LS^{(-)}\sigma$ components [23], which are represented by the Pauli-spinor invariants

$$
\begin{aligned}
& \mathcal{O}^{LS} = i\boldsymbol{n}\cdot\boldsymbol{S}\ , \quad \mathcal{O}^{LS^{(-)}} = i\boldsymbol{n}\cdot\boldsymbol{S}^{(-)}\ , \quad \mathcal{O}^{LS^{(-)}\sigma} = i\boldsymbol{n}\cdot\boldsymbol{S}^{(-)}\,P_\sigma \quad , \\
& \text{with} \qquad \boldsymbol{n} = [\boldsymbol{k}^\prime_{12}\times\boldsymbol{k}_{12}]\ , \quad \boldsymbol{S} = \frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\ , \quad \boldsymbol{S}^{(-)} = \frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) \quad , \\
& \text{and} \qquad P_\sigma = \frac{1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{2} \quad .
\end{aligned} \tag{6}
$$

The invariant kernel $M^\Omega(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12})$ in Eq. (5) consists of various types of spin-flavor factors $X^\Omega_{\mathcal{T}}$ and the spatial functions $f^\Omega_{\mathcal{T}}(\theta)$ calculated for the quark-exchange kernel of the FB interaction. These are explicitly given in [23] and Appendix A. When the contribution from the exchange Feynman diagram is incorporated with the exchange operator $P_\sigma P_F P_r$, the total Born kernel is expanded as

$$
\begin{aligned}
& M(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) - M(\boldsymbol{k}_{12},-\boldsymbol{k}^\prime_{12})\,P_\sigma\,P_F \\
& = \sum_\Omega M^{\Omega\ \mathrm{total}}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12})\ \mathcal{O}^\Omega(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) \quad ,
\end{aligned} \tag{7}
$$

with the matrix element in the isospin basis

$$
\begin{aligned}
& \langle\,[BN]_{II_z}\,|\,M(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) - M(\boldsymbol{k}_{12},-\boldsymbol{k}^\prime_{12})\,P_\sigma\,P_F\,|\,[BN]_{II_z}\,\rangle \\
& = \sum_\Omega M^{\Omega\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12})\ \mathcal{O}^\Omega(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) \quad .
\end{aligned} \tag{8}
$$

Here the LS components $M^{\Omega\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12})$ for the NN and YN systems are explicitly given by

$$
\begin{aligned}
M^{LS\ \mathrm{total}}_{NN}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) &= \sum_{\mathcal{T}} (X^{LS}_{\mathcal{T}})_{NN}\left[\ f^{LS}_{\mathcal{T}}(\theta) - (-1)^I\,f^{LS}_{\mathcal{T}}(\pi-\theta)\ \right] \quad , \\
M^{LS\ \mathrm{total}}_{YY}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) &= \sum_{\mathcal{T}} \left[(X^{LS}_{\mathcal{T}})^{ud}_{YY}\,f^{LS}_{\mathcal{T}}(\theta) + (X^{LS}_{\mathcal{T}})^{s}_{YY}\,f^{LS}_{\mathcal{T}}(\pi-\theta)\ \right] \quad , \\
M^{LS^{(-)}\ \mathrm{total}}_{YY}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) &= \sum_{\mathcal{T}} \left[(X^{LS^{(-)}}_{\mathcal{T}})^{ud}_{YY}\,f^{LS}_{\mathcal{T}}(\theta)\right. \\
&\qquad \left. +(X^{LS^{(-)}\sigma}_{\mathcal{T}})^{s}_{YY}\,f^{LS}_{\mathcal{T}}(\pi-\theta)\ \right] \quad , \\
M^{LS^{(-)}\sigma\ \mathrm{total}}_{YY}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12}) &= \sum_{\mathcal{T}} \left[(X^{LS^{(-)}\sigma}_{\mathcal{T}})^{ud}_{YY}\,f^{LS}_{\mathcal{T}}(\theta)\right. \\
&\qquad \left. +(X^{LS^{(-)}}_{\mathcal{T}})^{s}_{YY}\,f^{LS}_{\mathcal{T}}(\pi-\theta)\ \right] \quad .
\end{aligned} \tag{9}
$$

We should note that the spin-flavor factors depend on isospin and the LS function $f^\Omega_{\mathcal{T}}(\theta)$ given in Eq. (A.3) is a function of $\boldsymbol{k}^2_{12}$ and $(\boldsymbol{k}^\prime_{12})^2$, in addition

to the relative angle: $\cos\theta = \widehat{\boldsymbol{k}}_{12} \cdot \widehat{\boldsymbol{k}'}_{12}$. The sum over $\mathcal{T}$ in Eq. (9) is with respect to the quark-exchange interaction types $\mathcal{T} = S,\ S'$ and $D_+,\ D_-$ [26], where the former two come from the aLS term of the FB interaction and the latter two from the sLS term (see Eq. (5.2) of [23]). For the YN system, the exchange term (i.e., $f_{\mathcal{T}}^{LS}(\pi-\theta)$ term) in Eq. (9) originates from the strangeness exchange process and the spin-flavor factors of the $LS^{(-)}$ and $LS^{(-)}\sigma$ types are interchanged between the $LS^{(-)}$ and $LS^{(-)}\sigma$ terms. The s.p. wave functions are expressed as

$$
\begin{aligned}
\psi(\boldsymbol{k}, s) &= \frac{1}{(2\pi)^3} \int d\,\boldsymbol{k}\ e^{-i\boldsymbol{k}\boldsymbol{r}} \psi(\boldsymbol{r}, s) \quad , \\
\psi(\boldsymbol{r}, s) &= \sum_{m_\ell m_s} \langle\, \ell m_\ell \tfrac{1}{2} m_s | jm \,\rangle\ \phi_{n\ell m_\ell}(\boldsymbol{r})\, \chi_{\frac{1}{2}m_s}(s) \quad , \\
\phi_{n\ell m_\ell}(\boldsymbol{r}) &= R_{n\ell}(r)\, Y_{\ell m_\ell}(\widehat{\boldsymbol{r}}) \quad ,
\end{aligned}
\tag{10}
$$

for the valence particle and the core nucleons; $v,\ c = n\ell j$. By noting

$$
\langle\, \boldsymbol{k}_1 \boldsymbol{k}_2\ [BN]^{II_z}\, |\, vc \,\rangle = \psi_v(\boldsymbol{k}_1, s_1)\, \psi_c(\boldsymbol{k}_2, s_2)\, \langle\ I_v\, I_{vz} \tfrac{1}{2}\ \tau\, |\, II_z \,\rangle \quad , \tag{11}
$$

with $\tau = 1/2$ for $c = p$ and $\tau = -1/2$ for $c = n$ and taking a sum over c for the core protons ($c_\tau = c_{1/2}$) and neutrons ($c_\tau = c_{-1/2}$) separately, Eq. (3) becomes $E_v^{s.s.} = \sum_\tau E_{v;\tau}^{s.s.}$ with

$$
\begin{aligned}
E_{v;\tau}^{s.s.} = &\sum_{c_\tau} \sum_I C_\tau^I(B)\ \frac{1}{(2\pi)^3} \sum_{\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}'_1, \boldsymbol{k}'_2} \psi_v^\dagger(\boldsymbol{k}_1, s_1)\, \psi_{c_\tau}^\dagger(\boldsymbol{k}_2, s_2)\ \delta(\boldsymbol{K}_{12} - \boldsymbol{K}'_{12}) \\
&\times \sum_\Omega M_{BB}^{\Omega\ \mathrm{total}}(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12})\, \mathcal{O}^\Omega(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12})\, \psi_v(\boldsymbol{k}'_1, s_1)\, \psi_{c_\tau}(\boldsymbol{k}'_2, s_2) \quad .
\end{aligned}
\tag{12}
$$

The isospin factor defined by

$$
C_\tau^I(B) = \sum_{I_z} \langle I_v\, I_{vz} \tfrac{1}{2} \tau\, |\, II_z \,\rangle^2 \tag{13}
$$

is given in Eq. (A.1). The implicit spin sum over s_2 is easily carried out:

$$
\begin{aligned}
&\sum_{c_\tau} \psi_{c_\tau}^\dagger(\boldsymbol{k}_2, s_2) \left\{ \begin{array}{c} \boldsymbol{S} \\ \boldsymbol{S}^{(-)} \end{array} \right\} \psi_{c_\tau}(\boldsymbol{k}'_2, s_2) = \frac{1}{2}\ \rho_\tau(\boldsymbol{k}_2, \boldsymbol{k}'_2)\ \boldsymbol{\sigma}_1 \quad , \\
&\sum_{c_\tau} \psi_{c_\tau}^\dagger(\boldsymbol{k}_2, s_2)\ \boldsymbol{S}^{(-)} P_\sigma\ \psi_v(\boldsymbol{k}'_1, s_1)\ \psi_{c_\tau}(\boldsymbol{k}'_2, s_2) = 0 \quad ,
\end{aligned}
\tag{14}
$$

where the core-density of the protons or neutrons are defined by

$$\rho_\tau(\boldsymbol{k}_2,\boldsymbol{k}_2^\prime)=2\sum_{(n\ell m_\ell)\,\in\, c_\tau}\phi^*_{n\ell m_\ell}(\boldsymbol{k}_2)\ \phi_{n\ell m_\ell}(\boldsymbol{k}_2^\prime)\quad . \tag{15}$$

We find that the $LS^{(-)}\sigma$ term does not contribute due to the spin-averaging. After all, we have obtained

$$\begin{aligned}
&E^{s.s.}_{v;\tau}=\sum_I C^I_\tau(B)\,\frac{1}{2(2\pi)^3}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_1^\prime,\boldsymbol{k}_2^\prime}\psi^\dagger_v(\boldsymbol{k}_1,s_1)\ \delta(\boldsymbol{K}_{12}-\boldsymbol{K}^\prime_{12})\ \rho_\tau(\boldsymbol{k}_2,\boldsymbol{k}_2^\prime)\\
&\times\left[\ M^{LS\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12})+M^{LS^{(-)}\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12})\ \right]\\
&\times i\,[\boldsymbol{k}^\prime_{12}\times\boldsymbol{k}_{12}]\cdot\boldsymbol{\sigma}_1\ \psi_v(\boldsymbol{k}_1^\prime,s_1)\quad .
\end{aligned}\tag{16}$$

So far we have made no approximation.

We can eliminate the $\boldsymbol{k}_2^\prime$ sum in Eq. (16) through $\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{k}_1^\prime+\boldsymbol{k}_2^\prime$; i.e., $\boldsymbol{k}_2^\prime=\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_1^\prime$. If we use the momenta $\boldsymbol{q}$ and $\boldsymbol{p}$ defined by

$$\begin{aligned}
&\boldsymbol{q}=\boldsymbol{k}_{12}-\boldsymbol{k}^\prime_{12}=\boldsymbol{k}_1-\boldsymbol{k}_1^\prime\quad ,\\
&\boldsymbol{p}=\boldsymbol{k}^\prime_{12}+\frac{1}{\xi}\boldsymbol{k}_{12}=\boldsymbol{k}_1^\prime-\frac{1}{\xi}\boldsymbol{k}_2\quad ,
\end{aligned}\tag{17}$$

the outer product $[\boldsymbol{k}^\prime_{12}\times\boldsymbol{k}_{12}]$ in Eq. (16) can be expressed as

$$[\boldsymbol{k}^\prime_{12}\times\boldsymbol{k}_{12}]=-\frac{\xi}{1+\xi}\left\{\ [\boldsymbol{k}_1\times\boldsymbol{k}_1^\prime]+\frac{1}{\xi}[\boldsymbol{k}_2\times\boldsymbol{q}]\ \right\}\quad . \tag{18}$$

The essential point of the Scheerbaum's discussion [16] is that his space integrals $D(q)/q$ and $E(p)/p$ are very smooth functions with respect to the small values of momentum transfers $q=|\boldsymbol{q}|$ and $p=|\boldsymbol{p}|$. From this observation, he replaced the integral by a constant value $\langle D(q)/q+E(p)/p\rangle$ evaluated at an appropriate averaged value $p=q=\bar{q}$ and carried out the summation over $\boldsymbol{k}_1,\ \boldsymbol{k}_2$ and $\boldsymbol{k}_1^\prime$ in Eq. (16). This behavior of the integral is related to the short-range character of the LS interaction and the assumption of the locality of the effective spin-orbit potential adopted there. Though the non-locality of the exchange kernel in the present case makes it difficult to follow his argument directly, we can make use of the short-range character of the LS force and assume that the amplitudes $M^{\Omega\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}^\prime_{12})$ in Eq. (16) have a very weak $\boldsymbol{k}_1,\ \boldsymbol{k}_2$ and $\boldsymbol{k}_1^\prime$ dependence. As we will see later, this assumption turns out to be fairly good even in our case. Following the same procedure as developed

by Scheerbaum, we can eventually arrive at the s.p. spin-orbit potential of the Thomas type:

$$
\begin{aligned}
E^{s.s.}_{v;\tau} &= \int d\,\boldsymbol{r}_1\,\psi^{\dagger}_{v}(\boldsymbol{r}_1,s_1)\ U_\tau(\boldsymbol{r}_1)\ \psi_v(\boldsymbol{r}_1,s_1) \quad , \\
U_\tau(\boldsymbol{r}) &= K_\tau\ \frac{1}{r}\frac{d\rho_\tau(r)}{dr}\ \boldsymbol{\ell}\cdot\boldsymbol{\sigma}_1 \quad , \qquad (19)
\end{aligned}
$$

where the proton ($\tau = 1/2$) and neutron ($\tau = -1/2$) densities are defined by

$$
\rho_\tau(r) = 2 \sum_{(n\ell)\,\in\, c_\tau} \frac{4\pi}{2\ell+1}[\,R_{n\ell}(r)\,]^2 \quad , \qquad (20)
$$

and the strength factor is given by

$$
\begin{aligned}
K_\tau = -\frac{1}{2}\ \frac{\xi}{1+\xi} & \sum_I C^I_\tau(B) \\
& \times \overline{\left[\,M^{LS\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}'_{12}) + M^{LS^{(-)}\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}'_{12})\,\right]} \quad . \qquad (21)
\end{aligned}
$$

For the s.s. nuclei with equal proton and neutron numbers (i.e., $Z = N$), the formulae in Eqs. (19) and (21) are further simplified into

$$
\begin{aligned}
U(\boldsymbol{r}) &= -\frac{\pi}{2}\ S_B\ \frac{1}{r}\frac{d\rho(r)}{dr}\ \boldsymbol{\ell}\cdot\boldsymbol{\sigma} \quad , \\
S_B &= \frac{1}{2\pi}\ \frac{\xi}{1+\xi} \sum_I \frac{2I+1}{2I_B+1} \\
& \times \overline{\left[\,M^{LS\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}'_{12}) + M^{LS^{(-)}\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12},\boldsymbol{k}'_{12})\,\right]} \quad , \qquad (22)
\end{aligned}
$$

where $\rho(r) = \rho_n(r) + \rho_p(r)$ with $\rho_n(r) = \rho_p(r)$ is the total density and the sum formula (A.2) for $C^I_\tau(B)$ is used. We call S_B in Eq. (22) the Scheerbaum factor.

2.2 Born approximation

Let us calculate the Scheerbaum factor for the s.s. symmetric nuclei in the Born approximation. For $B = N$ with $I_B = 1/2$, we have two possible isospin values $I = 0$ and 1 in Eq. (22). Then the invariant parts of the Born kernel in Eq. (9) yield

Table 1
The spin-flavor factors $X_{\mathcal{T}}^B$ as a function of $\lambda = (m_s/m_{ud})$. Note that $X_{S'}^B = X_S^B$. The $X_{\mathcal{T}}^B$ values for $\lambda = 1$ are given in the second line. The last row implies off-diagonal factors for the ΛN-ΣN coupling.

B	$X_{D_-}^B$	$X_{D_+}^B$	X_S^B
N	$\frac{14}{9}$	$-\frac{10}{27}$	$\frac{16}{81}$
Λ	$\frac{2}{9\lambda}\left(2+\frac{1}{\lambda}\right)$	$-\frac{1}{9\lambda}\left(2+\frac{1}{\lambda}\right)$	$\frac{1}{18\lambda}\left(2-\frac{1}{\lambda}\right)$
$\lambda = 1$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{18}$
Σ	$\frac{2}{3\cdot 81}\left(106-\frac{6}{\lambda}-\frac{1}{\lambda^2}\right)$	$-\frac{1}{81}\left(18-\frac{10}{\lambda}-\frac{3}{\lambda^2}\right)$	$\frac{1}{6\cdot 81}\left(26+\frac{42}{\lambda}-\frac{7}{\lambda^2}\right)$
$\lambda = 1$	$\frac{22}{27}$	$-\frac{5}{81}$	$\frac{61}{6\cdot 81}$
Ξ	$-\frac{2}{9}$	$-\frac{1}{81}\left(1+\frac{14}{\lambda}+\frac{6}{\lambda^2}\right)$	$-\frac{1}{2\cdot 81}\left(1+\frac{18}{\lambda}-\frac{6}{\lambda^2}\right)$
$\lambda = 1$	$-\frac{2}{9}$	$-\frac{7}{27}$	$-\frac{13}{2\cdot 81}$
$\begin{pmatrix}\Lambda\text{-}\Sigma \\ \Sigma\text{-}\Lambda\end{pmatrix}$	$-\frac{2}{27}\left(7+\frac{2}{\lambda}\right)$ $-\frac{2}{3}$	$-\frac{1}{81}\left(5-\frac{2}{\lambda}\right)$ $-\frac{1}{27}$	$-\frac{1}{2\cdot 81}\left(13+\frac{6}{\lambda}\right)$ $-\frac{19}{2\cdot 81}$

$$S_N = \frac{1}{8\pi}\left\{ \sum_{\mathcal{T}} (X_{\mathcal{T}}^{LS})_{NN}^{I=0} \left[\overline{f_{\mathcal{T}}^{LS}(\theta) - f_{\mathcal{T}}^{LS}(\pi-\theta)}\right] \right.$$
$$\left. +3\sum_{\mathcal{T}} (X_{\mathcal{T}}^{LS})_{NN}^{I=1} \left[\overline{f_{\mathcal{T}}^{LS}(\theta) + f_{\mathcal{T}}^{LS}(\pi-\theta)}\right]\right\} \quad . \tag{23}$$

Here $I = 0$ corresponds to the ^3E state and $I = 1$ to the ^3O state. If we assume

$$\overline{f_{\mathcal{T}}^{LS}(\theta)} = \overline{f_{\mathcal{T}}^{LS}(\pi-\theta)} \quad , \tag{24}$$

we find that only the ^3O state contributes to S_N and obtain

$$S_N = \frac{3}{4\pi}\sum_{\mathcal{T}} (X_{\mathcal{T}}^{LS})_{NN}^{I=1}\ \overline{f_{\mathcal{T}}^{LS}(\theta)} \quad . \tag{25}$$

This expression corresponds to Scheerbaum's Eq. (3.57) [16]. Thus we have a correspondence

$$\sum_{\mathcal{T}} (X_{\mathcal{T}}^{LS})_{NN}^{I=1}\ \overline{f_{\mathcal{T}}^{LS}(\theta)} \sim \frac{4\pi}{\bar{q}} \int_0^\infty s^3\ j_1(\bar{q}s)\ g^{^3O}(s)\ d\,s \quad . \tag{26}$$

Under the same assumption as Eq. (24) the Scheerbaum factors for the hyperons are given by

$$S_B = \frac{1}{2\pi}\,\frac{\xi}{1+\xi}\sum_{\mathcal{T}} X^B_{\mathcal{T}}\,\overline{f^{LS}_{\mathcal{T}}(\theta)} \quad , \tag{27}$$

where $X^B_{\mathcal{T}}$ for $B=Y$ are defined by

$$X^B_{\mathcal{T}} = \sum_I \frac{2I+1}{2I_B+1}\left[(X^{LS}_{\mathcal{T}})^{ud}_{BB} + (X^{LS}_{\mathcal{T}})^{s}_{BB} + (X^{LS^{(-)}}_{\mathcal{T}})^{ud}_{BB} + (X^{LS^{(-)}\sigma}_{\mathcal{T}})^{s}_{BB}\right] . \tag{28}$$

In this notation, $X^N_{\mathcal{T}}$ is given by (ξ=1)

$$X^N_{\mathcal{T}} = 3\ (X^{LS}_{\mathcal{T}})^{I=1}_{NN} = 3\ (X^{LS}_{\mathcal{T}})^{^3O}_{NN} \quad . \tag{29}$$

The spin-flavor factors $X^B_{\mathcal{T}}$ can be easily obtained from the explicit expressions of $X^{LS}_{\mathcal{T}}$, $X^{LS^{(-)}}_{\mathcal{T}}$ and $X^{LS^{(-)}\sigma}_{\mathcal{T}}$, which are given in Appendix C of [23]. [6] They are tabulated in Table 1. When we derive these results, we should note that $\mathcal{P}'=1$ in

$$(X^{LS}_{\mathcal{T}})_{a\mathcal{P},a'\mathcal{P}'} = (X^{LS}_{\mathcal{T}})^{ud}_{aa'} + (X^{LS}_{\mathcal{T}})^{s}_{aa'}\ \mathcal{P}' \tag{30}$$

corresponds to the ^3O contribution. For $B=\Sigma$, the isospin sum in Eq. (28) gives

$$\begin{aligned}
\text{isoscalar term} &= \sum_I \frac{2I+1}{2I_B+1}\cdot 1 = \frac{1}{2I_B+1}\sum_{II_z} 1 \\
&= \frac{1}{2I_B+1}\sum_{I_{Bz}} 1 \sum_{\tau} 1 = 2 \quad , \\
\text{isovector term} &= \sum_I \frac{2I+1}{2I_B+1}\ (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)_I = \frac{1}{2I_B+1}\sum_{II_z}\langle\, II_z\,|\,\boldsymbol{\tau}_B\cdot\boldsymbol{\tau}_N\,|\,II_z\,\rangle \\
&= \frac{1}{2I_B+1}\ (Tr\,\boldsymbol{\tau}_B)\ (Tr\,\boldsymbol{\tau}_N) = 0 \quad .
\end{aligned} \tag{31}$$

The factor 2 for the isoscalar term is the sum over the proton and neutron, and the isovector term does not contribute since we have assumed $Z=N$ (the total isospin is zero for α, ^{16}O and ^{40}Ca).

[6] $\lambda = m_{ud}/m_s$ in Appendix C of [23] is a misprint of $\lambda = m_s/m_{ud}$.

Table 2
Quark-model parameters

model	b (fm)	m_{ud} (MeV/c^2)	α_S	$\lambda = m_s/m_{ud}$
RGM-F	0.6	313	1.5187	1.25
FSS	0.616	360	2.1742	1.526
RGM-H	0.667	389	2.1680	1.490

Next we discuss the averaging procedure in $\overline{f_{\mathcal{T}}^{LS}(\theta)}$. A possible approximation to obtain $\overline{f_{\mathcal{T}}^{LS}(\theta)}$ is to use the Wigner transform $G_W^{LS}(\boldsymbol{r}, \boldsymbol{p})$ (which is given in Eq. (2.16) of [23]) at $\boldsymbol{p} = 0$, and to follow the Scheerbaum's prescription for the local potential $G_W^{LS}(\boldsymbol{r}, 0)$. We can show that this procedure is equivalent to set $\boldsymbol{q} = 0$ in Eq. (A.3). In this case the θ-dependence in $f_{\mathcal{T}}^{LS}(\theta)$ disappears ($\theta = \pi$) and $\overline{f_{\mathcal{T}}^{LS}(\theta)}$ becomes a function of the momentum transfer $k = |\boldsymbol{k}|$ ($\boldsymbol{k} = \boldsymbol{q}_f - \boldsymbol{q}_i$). This corresponds to the Scheerbaum's parameter $\overline{q}$. We call this the Scheerbaum approximation.

The relationship between the basic Born kernel in Eq. (5) and the Wigner transform for the LS component is given by

$$\begin{aligned} M(\boldsymbol{q}_f, \boldsymbol{q}_i) &= \int d\boldsymbol{r}\ e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ G_W(\boldsymbol{r}, \boldsymbol{q}) \\ &= \sum_{\mathcal{T}} X_{\mathcal{T}}^{LS} \int d\boldsymbol{r}\ e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ G_{W\mathcal{T}}^{LS}(\boldsymbol{r}, \boldsymbol{q})\ [\,\boldsymbol{r} \times \boldsymbol{q}\,] \cdot \boldsymbol{S}\ , \end{aligned} \tag{32}$$

where $\boldsymbol{k} = \boldsymbol{q}_f - \boldsymbol{q}_i$ and $\boldsymbol{q} = (1/2)(\boldsymbol{q}_f + \boldsymbol{q}_i)$. The spatial function $G_{W\mathcal{T}}^{LS}(\boldsymbol{r}, \boldsymbol{q} = 0)$ becomes a function of $r = |\boldsymbol{r}|$ only, and thus we can carry out the angle integral $\int d\hat{\boldsymbol{r}}$ after the partial wave expansion of the plane wave. Then the component with the angular momentum $\ell = 1$ only survives, and we obtain

$$\begin{aligned} & M(\boldsymbol{q}_f, \boldsymbol{q}_i)|_{\boldsymbol{q}=0} \\ & = \sum_{\mathcal{T}} X_{\mathcal{T}}^{LS}\ \frac{4\pi}{k} \int_0^\infty r^3\ dr\ j_1(kr)\ G_{W\mathcal{T}}^{LS}(\boldsymbol{r}, 0)\ \mathcal{O}^{LS}(\boldsymbol{q}_f, \boldsymbol{q}_i), \end{aligned} \tag{33}$$

or

$$f_{\mathcal{T}}^{LS}(\theta)|_{\boldsymbol{q}=0} = \frac{4\pi}{k} \int_0^\infty r^3\ dr\ j_1(kr)\ G_{W\mathcal{T}}^{LS}(\boldsymbol{r}, 0)\ \ . \tag{34}$$

Note that $|_{\boldsymbol{q}=0}$ implies setting $\boldsymbol{q}=0$ except for the LS operator part. If we call $G_W^{LS}(\boldsymbol{r},0)=\sum_{\mathcal{T}} X_{\mathcal{T}}^{LS} G_{W\mathcal{T}}^{LS}(\boldsymbol{r},0)$ the LS potential, we find

$$\sum_{\mathcal{T}} X_{\mathcal{T}}^{LS} f_{\mathcal{T}}^{LS}(\theta)|_{\boldsymbol{q}=0} = \frac{4\pi}{k} \int_0^\infty r^3\, d\,r\; j_1(kr)\; G_W^{LS}(\boldsymbol{r},0) \quad , \tag{35}$$

which is nothing but Eq. (26) if we assign

$$G_W^{LS}(\boldsymbol{r},0) \sim g^{^3O}(r) \quad . \tag{36}$$

Then we find

$$\overline{f_{\mathcal{T}}^{LS}(\theta)} \sim f_{\mathcal{T}}^{LS}(\theta)|_{\boldsymbol{q}=0} \quad , \tag{37}$$

with $k=\overline{q}$ (see Eq. (A.5)). We will discuss the choice of the value $k=\overline{q}$ and a further simplification in the next section,

Another approximation for the LS function $f_{\mathcal{T}}^{LS}(\theta)$ in Eq. (27) is obtained by taking only P-wave components in the partial wave expansion of the Born kernel [24]. Suppose the partial wave expansion of Eq. (8) is

$$\begin{aligned} & \sum_{\Omega} M_{aa'}^{\Omega\ \mathrm{total}}(\boldsymbol{q}_f,\boldsymbol{q}_i)\ \mathcal{O}^{\Omega}(\boldsymbol{q}_f,\boldsymbol{q}_i) \\ & = \sqrt{(1+\delta_{a_1,a_2})(1+\delta_{a_1',a_2'})} \sum_{JM\ell\ell' SS'} 4\pi\ R_{\alpha S\ell,\alpha' S'\ell'}^{\Omega\ J}(q_f,q_i) \\ & \times \mathcal{Y}_{(\ell S)JM}(\hat{\boldsymbol{q}}_f; spin)\ \mathcal{Y}_{(\ell' S')JM}^{*}(\hat{\boldsymbol{q}}_i; spin) \quad , \end{aligned} \tag{38}$$

where $\mathcal{Y}_{(\ell S)JM}(\hat{\boldsymbol{q}}; spin)=[Y_\ell(\hat{\boldsymbol{q}})\chi_S(spin)]_{JM}$ is the standard space-spin wave function. The front factor $\sqrt{(1+\delta_{a_1,a_2})(1+\delta_{a_1',a_2'})}$ is 2 for NN and 1 for YN. The partial-wave amplitudes $R_{\alpha S\ell,\alpha' S'\ell'}^{\Omega\ J}(q_f,q_i)$ for $\Omega=LS$ and $LS^{(-)}$ are explicitly given by

$$\begin{aligned} & R_{\alpha S\ell,\alpha' S'\ell'}^{LS\ J}(q_f,q_i) = \delta_{\ell,\ell'}\ \delta_{S,S'}\ \delta_{S,1}\ q_f q_i\ \frac{1}{2(2\ell+1)}\ [\ell(\ell+1)+2-J(J+1)] \\ & \times \sum_{\mathcal{T}} \left(X_{\mathcal{T}}^{LS}\right)_{\alpha\alpha'} \left(f_{\mathcal{T}\ \ell+1}^{LS} - f_{\mathcal{T}\ \ell-1}^{LS}\right) \quad , \\ & R_{\alpha S\ell,\alpha' S'\ell'}^{LS^{(-)}\ J}(q_f,q_i) = \delta_{\ell,\ell'}\ \delta_{J,\ell}\ q_f q_i \frac{\sqrt{J(J+1)}}{2J+1} \sum_{\mathcal{T}} \Big[\ \left(X_{\mathcal{T}}^{LS^{(-)}}\right)_{\alpha\alpha'} \\ & + \left(X_{\mathcal{T}}^{LS^{(-)}\sigma}\right)_{\alpha\alpha'}\ (-1)^{1-S'}\ \Big]\ \left(f_{\mathcal{T}\ \ell-1}^{LS} - f_{\mathcal{T}\ \ell+1}^{LS}\right) \\ & \qquad (S,S'=1,0 \text{ or } 0,1 \text{ only}) \ , \end{aligned} \tag{39}$$

where

$$f^{LS}_{\mathcal{T}\ell} = \frac{1}{2} \int_0^\pi f^{LS}_{\mathcal{T}}(\theta)\ P_\ell(\cos\theta)\ \sin\theta d\ \theta \tag{40}$$

is the angular-momentum projection of the LS function Eq. (A.3). (See Eq. (2.29) of [24].) If we take $\ell = \ell' = 1$ only in Eq. (38) and use the formulae ($\boldsymbol{n} = [\boldsymbol{q}_i \times \boldsymbol{q}_f]$)

$$\begin{aligned}
\mathcal{O}^{LS} &= i\boldsymbol{n} \cdot \boldsymbol{S} = -\frac{2\pi}{3} q_f q_i \sum_{JM} [4 - J(J+1)] \\
&\quad \times \mathcal{Y}_{(11)JM}(\hat{\boldsymbol{q}}_f; spin)\ \mathcal{Y}^*_{(11)JM}(\hat{\boldsymbol{q}}_i; spin)\quad , \\
\mathcal{O}^{LS(-)} &= i\boldsymbol{n} \cdot \boldsymbol{S}^{(-)} = 4\pi \frac{\sqrt{2}}{3} q_f q_i \sum_M \Big\{\ \mathcal{Y}_{(11)1M}(\hat{\boldsymbol{q}}_f; spin)\ \mathcal{Y}^*_{(10)1M}(\hat{\boldsymbol{q}}_i; spin) \\
&\quad + \mathcal{Y}_{(10)1M}(\hat{\boldsymbol{q}}_f; spin)\ \mathcal{Y}^*_{(11)1M}(\hat{\boldsymbol{q}}_i; spin) \Big\}\quad , \\
\mathcal{O}^{LS(-)\sigma} &= i\boldsymbol{n} \cdot \boldsymbol{S}^{(-)} P_\sigma = 4\pi \frac{\sqrt{2}}{3} q_f q_i \sum_M \Big\{\ -\mathcal{Y}_{(11)1M}(\hat{\boldsymbol{q}}_f; spin)\ \mathcal{Y}^*_{(10)1M}(\hat{\boldsymbol{q}}_i; spin) \\
&\quad + \mathcal{Y}_{(10)1M}(\hat{\boldsymbol{q}}_f; spin)\ \mathcal{Y}^*_{(11)1M}(\hat{\boldsymbol{q}}_i; spin) \Big\}\quad ,
\end{aligned} \tag{41}$$

we eventually find that this prescription corresponds to the approximation [7]

$$\begin{aligned}
M^{LS\ \mathrm{total}}_{aa'}(\boldsymbol{q}_f, \boldsymbol{q}_i) &\sim \begin{cases} 2 \sum_{\mathcal{T}} \left(X^{LS}_{\mathcal{T}}\right)^{I=1}_{NN} \left(f^{LS}_{\mathcal{T}0} - f^{LS}_{\mathcal{T}2}\right) \\ \sum_{\mathcal{T}} \left[\ \left(X^{LS}_{\mathcal{T}}\right)^{ud}_{aa'} + \left(X^{LS}_{\mathcal{T}}\right)^{s}_{aa'}\ \right]\ \left(f^{LS}_{\mathcal{T}0} - f^{LS}_{\mathcal{T}2}\right) \end{cases} \\
&\qquad \text{for} \quad \begin{cases} NN \\ YN \end{cases} \quad , \\
M^{LS(-)\ \mathrm{total}}_{aa'}(\boldsymbol{q}_f, \boldsymbol{q}_i) &\sim \sum_{\mathcal{T}} \left[\ \left(X^{LS(-)}_{\mathcal{T}}\right)^{ud}_{aa'} + \left(X^{LS(-)\sigma}_{\mathcal{T}}\right)^{s}_{aa'}\ \right] \\
&\qquad \times \left(f^{LS}_{\mathcal{T}0} - f^{LS}_{\mathcal{T}2}\right) \qquad \text{for} \quad YN \quad .
\end{aligned} \tag{42}$$

If we compare these with Eq. (9), we find that this approximation corresponds to setting

$$\overline{f^{LS}_{\mathcal{T}}(\theta)} \sim f^{LS}_{\mathcal{T}0} - f^{LS}_{\mathcal{T}2} \quad . \tag{43}$$

Note that we still have two parameters $q_f = |\boldsymbol{q}_f|$ and $q_i = |\boldsymbol{q}_i|$, the choice of which will be discussed in the next section.

[7] Note that $\mathcal{O}^{LS(-)\sigma}$ part disappears because of the second equation of Eq. (14).

2.3 Realistic calculation

Here we consider a realistic calculation of S_B, based on the G-matrices for the NN and YN interaction [25]. In this case, $M^{\Omega\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12})$ in Eq. (16) now depends on some averaged value K of $\sqrt{(\boldsymbol{K}_{12})^2}$ and the starting energy $\omega = E_B(k'_1) + E_N(k'_2)$; i.e., $G^{\Omega\ \mathrm{total}}_{BB}(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12}; K, \omega)$. We assume the density $\rho_\tau(\boldsymbol{k}_2, \boldsymbol{k}'_2)$ in Eq. (15) is well approximated by the local form

$$\rho_\tau(\boldsymbol{k}_2, \boldsymbol{k}'_2) = \delta(\boldsymbol{k}_2 - \boldsymbol{k}'_2)\ \rho_\tau(\boldsymbol{k}_2) \quad , \tag{44}$$

where $\rho_\tau(\boldsymbol{k})$ is given by the Fermi distribution

$$\rho_\tau(\boldsymbol{k}) = 2\ \Theta(k^\tau_F - |\boldsymbol{k}|) \quad , \tag{45}$$

with Θ being the Heaviside's step function. Under this assumption, we can approximately set $\boldsymbol{k}_2 \sim \boldsymbol{k}'_2 = \boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}'_1$. This implies $\boldsymbol{k}_1 = \boldsymbol{k}'_1$ and $\boldsymbol{k}_2 = \boldsymbol{k}'_2$. This approximation, however, makes the ℓs factor in Eq. (18) disappear. Thus we must take the density average by Eqs. (44) and (45) only for the invariant part just as in Eq. (21) and (22). After the change of the notation $\boldsymbol{k}_2 \rightarrow \boldsymbol{q}_2$ and $\boldsymbol{k}_{12} \rightarrow \boldsymbol{q}$ *etc.*, the expression we use is

$$\begin{aligned} \overline{G^\Omega_{BB}(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12}; K, \omega)} &= \frac{\int \rho_\tau(\boldsymbol{q}_2)\ G^\Omega_{BB}(\boldsymbol{q}, \boldsymbol{q}'; K, \omega)\ d\,\boldsymbol{q}_2}{\int \rho_\tau(\boldsymbol{q}_2)\ d\,\boldsymbol{q}_2} \\ &= \frac{3}{4\pi}\ \frac{1}{(k_F)^3} \int_{|\boldsymbol{q}_2|<k_F} G^\Omega_{BB}(\boldsymbol{q}, \boldsymbol{q}'; K, \omega)\ d\,\boldsymbol{q}_2 \quad , \end{aligned} \tag{46}$$

where we have assumed symmetric nuclear matter and used $\rho_\tau(\boldsymbol{q}_2) \rightarrow \Theta(k_F - |\boldsymbol{q}_2|)$. Here we make use of the same treatment of angle-averaging just as used for the derivation of the s.p. potentials in [25]. We first change the integral

Table 3
Contributions of symmetric (sLS) and antisymmetric (aLS) LS terms of the FB interaction [23] to the nucleon Scheerbaum factor S_N in the simplest approximation with $\bar{q} = 0$. The unit is MeV $\cdot$ fm^5.

model	sLS	aLS	S_N	aLS/sLS
	$D_- + D_+$	$S + S'$	total	ratio
RGM-F	-28.2	-9.7	-37.8	0.344
FSS	-32.2	-11.0	-43.2	0.342
RGM-H	-32.2	-11.0	-43.2	0.342

Table 4
The Scheerbaum factor S_B in the simplest $\bar{q} = 0$ approximation by the $\boldsymbol{p} = 0$ Wigner transform $G_W(0)$. In $S_{\Lambda\text{-}\Sigma} = S_{\Sigma\text{-}\Lambda}$, the average mass of Λ and Σ is used for ξ. The unit is $\text{MeV} \cdot \text{fm}^5$. $\lambda = (m_s/m_{ud})$ implies the FSB. When $\lambda = 1$, we also assume $\xi = 1$.

B	RGM-F		FSS		RGM-H	
	$\lambda = 1$	$\lambda = 1.25$	$\lambda = 1$	$\lambda = 1.526$	$\lambda = 1$	$\lambda = 1.490$
N	-37.8		-43.2		-43.2	
Λ	-12.4	-9.0	-14.1	-8.3	-14.1	-8.6
Σ	-22.7	-19.3	-26.0	-21.5	-26.0	-21.5
Ξ	12.3	9.2	14.0	9.5	14.0	9.7
$S_{\Lambda\text{-}\Sigma}$	20.5	16.8	23.4	18.5	23.5	18.6

variable $\boldsymbol{q}_2$ in Eq. (46) to $\boldsymbol{q}$ through $\boldsymbol{q}_2 = \xi \boldsymbol{q}_1 - (1+\xi)\boldsymbol{q}$. We assume q_1 and determine K through $K = \sqrt{\overline{(\boldsymbol{K}_{12})^2}(q_1, q)}$. Since $q_2 = |\boldsymbol{q}_2|$ is given by q_1 and K, it is also determined from q_1 and q. The starting energy ω is determined as $\omega = E_B(q_1) + E_N(q_2)$. Then by using the weight function $W(q_1, q)$ introduced in Eq. (21) of [25], we find

$$\overline{G^{\Omega}_{BB}(\boldsymbol{k}_{12}, \boldsymbol{k}'_{12}; K, \omega)} = \frac{3}{4\pi} \frac{1}{(k_F)^3} (1+\xi)^3 \int_0^{q_{max}} q^2 \, dq \, W(q_1, q) \times \int d\widehat{\boldsymbol{q}} \, G^{\Omega}_{BB}(\boldsymbol{q}, \boldsymbol{q}; K, \omega) \quad . \tag{47}$$

In order to reduce the angular integral in Eq. (47) further, we need explicit formulae for the partial-wave decomposition of the invariant amplitudes.[8] For the LS and $LS^{(-)}$ components, these are given by $M^{LS\ \text{total}}_{aa'}(\boldsymbol{q}_f, \boldsymbol{q}_i) = (2/|\boldsymbol{n}|)\ h^0_{aa'}$ and $M^{LS^{(-)}\ \text{total}}_{aa'}(\boldsymbol{q}_f, \boldsymbol{q}_i) = (2/|\boldsymbol{n}|)\ h^-_{aa'}$, where $h^0_{aa'}$ and $h^-_{aa'}$ are the flavor matrix elements of

$$\begin{aligned} h^0 &= -\frac{1}{4} \sum_J \frac{(2J+1)}{J(J+1)} \Big[\, G^J_{1J,1J} \, P^1_J(\cos\theta) \\ &-(J+1)\, G^J_{1\,J-1,\,1\,J-1} \, P^1_{J-1}(\cos\theta) + J\, G^J_{1\,J+1,\,1\,J+1} \, P^1_{J+1}(\cos\theta) \Big] \quad , \\ h^- &= \frac{1}{4} \sum_J \frac{2J+1}{\sqrt{J(J+1)}} \Big[\, G^J_{1J,0J} + G^J_{0J,1J} \Big] \, P^1_J(\cos\theta) \quad . \end{aligned} \tag{48}$$

[8] The full expression of the partial-wave decomposition of the invariant amplitudes is given in Appendix D of [24].

Table 5
LS and $LS^{(-)} + LS^{(-)}\sigma$ contributions to the Scheerbaum factor S_Λ in the simplest approximation of Eq. (52). The model is FSS. $\lambda = (m_s/m_{ud})$ implies the FSB. When $\lambda = 1$, we also assume $\xi = 1$.

	$X^\Lambda_{D_-}$	$X^\Lambda_{D_+}$	X^Λ_S	$\widetilde{S}_\Lambda$	S_Λ
($\lambda = 1$)					
LS	$\frac{8}{9}$	$-\frac{2}{9}$	$\frac{1}{9}$	0.998	-24.4
$LS^{(-)} + LS^{(-)}\sigma$	$-\frac{2}{9}$	$-\frac{1}{9}$	$-\frac{1}{18}$	-0.421	10.3
sum	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{18}$	0.577	-14.1
($\lambda = 1.526$)					
LS	0.6723	-0.1395	0.1014	0.813	-18.2
$LS^{(-)} + LS^{(-)}\sigma$	-0.2856	-0.0539	-0.0525	-0.440	9.9
sum	0.3867	-0.1934	0.0489	0.373	-8.3

Here $G^J_{S\ell,S'\ell'} = G^J_{S\ell,S'\ell'}(q_f, q_i; K, \omega)$ and $P^1_J(\cos\theta)$ is the associated Legendre function of the first kind with degree 1. We set $\boldsymbol{q}_f = \boldsymbol{q}_i = \boldsymbol{q}$ and $\theta = 0$, and use $(1/\sin\theta)P^1_J(\cos\theta) = P_J{}'(\cos\theta)$ and $P_\ell{}'(1) = \ell(\ell+1)/2$. Then we easily find

$$
\begin{aligned}
& G^{LS}(\boldsymbol{q}, \boldsymbol{q}; K, \omega) = -\frac{1}{4q^2} \sum_{\ell=1}^{\infty} \Big[\ (2\ell-1)(\ell+1)\ G^{\ell-1}_{1\ell,1\ell} \\
& +(2\ell+1)\ G^{\ell}_{1\ell,1\ell} - (2\ell+3)\ell\ G^{\ell+1}_{1\ell,1\ell}\ \Big] \quad , \\
& G^{LS^{(-)}}(\boldsymbol{q}, \boldsymbol{q}; K, \omega) = \frac{1}{4q^2} \sum_{\ell=1}^{\infty} (2\ell+1)\sqrt{\ell(\ell+1)}\ \Big[\ G^{\ell}_{1\ell,0\ell} + G^{\ell}_{0\ell,1\ell}\ \Big] . \qquad (49)
\end{aligned}
$$

Combining Eqs. (22), (47) and (49), we obtain

$$
\begin{aligned}
& S_B(q_1) = -(1+\delta_{B,N})\ \frac{1}{2\pi}\ \frac{3}{4(k_F)^3}\ \xi(1+\xi)^2 \sum_{I,J} \frac{2I+1}{2I_B+1}(2J+1) \\
& \times \int_0^{q_{max}} d\,q\ W(q_1, q)\ \Big\{\ (J+2)G^J_{B1\,J+1,\,B1\,J+1}(q, q; K, \omega) \\
& +G^J_{B1J,B1J}(q, q; K, \omega) - (J-1)G^J_{B1\,J-1,\,B1\,J-1}(q, q; K, \omega) \\
& -\sqrt{J(J+1)}\ \Big[\ G^J_{B1J,B0J}(q, q; K, \omega) + G^J_{B0J,B1J}(q, q; K, \omega)\Big]\Big\} \quad . \qquad (50)
\end{aligned}
$$

3 Result and discussion

For the value of $k = \overline{q}$ in Eq. (37), we follow the suggestion by Scheerbaum [16] and take the value corresponding to the "wavelength" of the density distribution. If we take this to be $\sim 4t$ with $t \sim 2.4$ fm being the nuclear surface thickness, we arrive at the estimate [9]

$$k = \overline{q} \sim \frac{2\pi}{4t} \sim 0.7 \ \mathrm{fm}^{-1} \quad . \tag{51}$$

The simplest approximation is to set $k = \overline{q} = 0$ in Eq. (A.5). In this case we can write down an analytic expression for S_B :

$$\begin{aligned}
S_B &= -\alpha_S\, x^3\, m_{ud}c^2\, b^5\, \frac{\xi}{1+\xi}\, \widetilde{S}_B \quad , \\
\widetilde{S}_N &= \frac{14}{9} - \frac{10}{27}\left(\frac{3}{4}\right)^{\frac{3}{2}} + \frac{32}{81}\left(\frac{12}{11}\right)^{\frac{3}{2}} \quad , \\
\widetilde{S}_\Lambda &= \frac{2}{9\lambda}\left(2+\frac{1}{\lambda}\right) - \frac{1}{9\lambda}\left(2+\frac{1}{\lambda}\right)\left(\frac{3}{4}\right)^{\frac{3}{2}} + \frac{1}{9\lambda}\left(2-\frac{1}{\lambda}\right)\left(\frac{12}{11}\right)^{\frac{3}{2}} \quad , \\
\widetilde{S}_\Sigma &= \frac{2}{3\cdot 81}\left(106 - \frac{6}{\lambda} - \frac{1}{\lambda^2}\right) - \frac{1}{81}\left(18 - \frac{10}{\lambda} - \frac{3}{\lambda^2}\right)\left(\frac{3}{4}\right)^{\frac{3}{2}} \\
&\quad + \frac{1}{3\cdot 81}\left(26 + \frac{24}{\lambda} - \frac{7}{\lambda^2}\right)\left(\frac{12}{11}\right)^{\frac{3}{2}} \quad , \\
\widetilde{S}_\Xi &= -\frac{2}{9} - \frac{1}{81}\left(1 + \frac{14}{\lambda} + \frac{6}{\lambda^2}\right)\left(\frac{3}{4}\right)^{\frac{3}{2}} - \frac{1}{81}\left(1 + \frac{18}{\lambda} - \frac{6}{\lambda^2}\right)\left(\frac{12}{11}\right)^{\frac{3}{2}} \quad , \\
\widetilde{S}_{\Lambda\text{-}\Sigma} &= -\frac{2}{27}\left(7+\frac{2}{\lambda}\right) - \frac{1}{81}\left(5-\frac{2}{\lambda}\right)\left(\frac{3}{4}\right)^{\frac{3}{2}} - \frac{1}{81}\left(13+\frac{6}{\lambda}\right)\left(\frac{12}{11}\right)^{\frac{3}{2}} ,
\end{aligned} \tag{52}$$

where $x = (\hbar/m_{ud}cb)$, $\lambda = (m_s/m_{ud})$ and $\xi = (M_N/M_B)$. The value of S_B in this simplest approximation is given in Table 4. The parameters of our three models, RGM-F, FSS and RGM-H, are given in Table 2. We note that FSS and RGM-H produce very similar results for the s.p. ℓs force, because the strength factor, $\alpha_S\, x^3\, m_{ud}c^2\, b^5\, \xi/(1+\xi)$, and the quark-mass ratio, $\lambda = (m_s/m_{ud})$, are very similar to each other. In particular, $\sqrt{2/\pi}\,\alpha_S\, x^3\, m_{ud}c^2$ is constrained to be 440 MeV for RGM-F and FSS, in order to reproduce the N-Δ mass splitting through the color-magnetic term of the FB interaction. The S_N $(= S_N^{3O})$ value, ~ -40 MeV $\cdot$ fm^5, is rather small, compared with the S_{free}^{3O} value, $-53 \sim -61$ MeV$\cdot$fm^5, given in Table 1 of the Scheerbaum's paper [16] for the Reid soft-core and other potentials. In his calculation, the effect

[9] This is almost half of the Fermi momentum $k_F = (9\pi/8)^{1/3}/r_0 = 1.36$ fm^{-1} for $r_0 = 1.12$ fm, which corresponds to $\rho = (3/4\pi)/{r_0}^3 = 0.170$ fm^{-3}.

Table 6
The Scheerbaum factors S_B predicted by FSS in various types of approximations. 1) $G_W(0)$: Born approximation with $\boldsymbol{p}=0$ Wigner transform with $\bar{q}=0$, 2) G_W: Born approximation with $\boldsymbol{p}=0$ Wigner transform with $\bar{q}=0.7$ fm^{-1}, 3) P: P-wave Born approximation with $q_f=q_i=0.35$ fm^{-1}, 4) G-matrix: QTQ or continuous choice with $q_1=0$ and $k_F=1.35$ fm^{-1}. The unit is MeV $\cdot$ fm^5.

B	Born			G-matrix		
	$G_W(0)$	G_W	P	QTQ	cont.	ratio
N	-43.2	-40.5	-41.7	-40.4	-41.6	1
Λ	-8.3	-7.8	-8.0	-3.8	-3.4	$\sim \frac{1}{12}$
Σ	-21.5	-20.1	-20.7	-27.5	-22.4	$\sim \frac{1}{2}$
Ξ	9.5	9.0	9.2			

of the short-range correlation reduces this value to $-34 \sim -47$ MeV $\cdot$ fm^5 in the same Table 1. These values were obtained with $\overline{q}=0.7$ fm^{-1} in the Scheerbaum approximation. However, we will see in the following that the effect of the short-range correlation obtained by solving the G-matrix equation is very small in our case. This is probably because our short-range repulsion is not represented by the hard core but by the quark-exchange kernel.

Table 3 shows that the Galilean non-invariant aLS term of the FB interaction [23] has a fairly large contribution to $S_N^{^3O}$. The magnitude of aLS contribution is almost 1/3 of the sLS contribution and they reinforce each other with the same sign [11].

We note that the S_Λ value changes significantly by the FSB, which is easily understood from the spin-flavor factors in Table 1. All the X_T^Λ factors contain the factor $1/\lambda$. If we assume $S_N^{^3O}=1$, then S_Λ is about 1/3 for $\lambda=1$, while it is $\sim 1/5$ for $\lambda=1.69$ (the maximum FSB). [10] On the other hand, S_Σ does not change very much by the FSB: it changes from 3/5 to 1/2 as λ changes from 1 to 1.69. The sign of S_Ξ is positive and its value changes from $-1/3$ to $-1/4$. The Λ-Σ coupling term is not small, and is about half of S_N both in $\lambda=1$ and $\lambda \neq 1$ cases. The sign of $S_{\Lambda\text{-}\Sigma}=S_{\Sigma\text{-}\Lambda}$ depends on the phase convention of the Λ and Σ flavor functions.

Table 5 shows the decomposition of S_Λ into LS and $LS^{(-)}+LS^{(-)}\sigma$ contributions in the simplest approximation. The signs of these two contributions are opposite to each other, and they largely cancel; namely, the half of the LS contribution is cancelled by the $LS^{(-)}+LS^{(-)}\sigma$ contribution. Because of this cancellation, the strong λ-dependence in S_Λ is even enhanced.

[10] When we set $\lambda=1$ in Eq. (52), we also neglect the mass difference of baryons; i.e., $\xi=1$.

Table 7
The nuclear-matter density dependence of the Scheerbaum factors S_B for N, Λ and Σ, predicted by quark-model G-matrices in the continuous prescription. The model is FSS. The unit is $\mathrm{MeV} \cdot \mathrm{fm}^5$.

k_F (fm^{-1})	1.07	1.20	1.35	
N	-43.0	-42.3	-41.3	(1)
Λ	-2.0	-2.7	-3.5	$(\sim \frac{1}{12})$
Σ	-21.5	-22.0	-21.8	$(\sim \frac{1}{2})$

Table 6 shows the predictions of S_B by FSS, calculated in the various prescriptions. The first column with $G_W(0)$ implies the simplest $\overline{q} = 0$ prescription, the second with G_W the Scheerbaum approximation with $\overline{q} = 0.7\ \mathrm{fm}^{-1}$, the third with the P-wave approximation of Eq. (43). In the last case, we have assumed $q_f = q_i = Q$ with $Q = \overline{q}/2 = 0.35\ \mathrm{fm}^{-1}$. We have examined the $\overline{q}$ or Q dependence in Eq. (37) or Eq. (43). Actually, the averaged spatial function $\overline{f^{LS}_T(\theta)}$ has some momentum dependence, so does S_B. However, this weak momentum dependence almost disappears if we take the ratio S_B/S_N. After all, we have found that S_B/S_N ratios in the Born approximation are approximately given by

$$\frac{S_\Lambda}{S_N} \sim \frac{1}{5} \quad , \quad \frac{S_\Sigma}{S_N} \sim \frac{1}{2} \quad , \quad \frac{S_\Xi}{S_N} \sim -\frac{1}{4} \quad , \tag{53}$$

independently of whichever approximation of the spatial functions is used. Table 6 also shows the results of the realistic calculation using G-matrix solutions in the QTQ and continuous prescriptions for intermediate spectra. Here the q_1 value in $S_B(q_1)$ (see Eq. (50)) is assumed to be $q_1 = 0$. We find that S_Λ receives a strong effect due to the short-range correlation and S_Λ/S_N is further reduced to almost 1/12. On the other hand, S_N and S_Σ do not change so much, except for the increase of $|S_\Sigma|$ in the QTQ prescription. The ratio, $S_\Sigma/S_N \sim 1/2$, does not seem to change very much even in the G-matrix calculation.

Table 7 shows the k_F dependence of $S_B(q_1 = 0)$ for N, Λ and Σ, which are calculated from the FSS G-matrices with the continuous choice. [11] The three values of the Fermi momentum, $k_F = 1.07,\ 1.2$ and $1.35\ \mathrm{fm}^{-1}$, correspond to the three densities of $\rho = 0.5\rho_0,\ 0.7\rho_0$ and ρ_0, respectively. Here $\rho_0 = 0.17\ \mathrm{fm}^{-3}$ is the normal density. We find that S_Λ/S_N becomes even smaller for lower densities, while S_Σ/S_N does not change much. Each contribution from the LS

[11] In [25] we assumed $U_B(q_1) = U_B(q_1 = 3.8\ \mathrm{fm}^{-1})$ for $q_1 \geq 3.8\ \mathrm{fm}^{-1}$, in order to avoid the unrealistic behavior [24] of the s.p. potentials in the high momentum region. The results in Tables 7 and 8 are obtained with this prescription, while those in Table 6 are without this prescription. The difference of S_B between these two prescriptions is very small, as is seen for $k_F = 1.35\ \mathrm{fm}^{-1}$.

Table 8
Decomposition of $S_\Lambda = -3.5$ MeV$\cdot$fm^5 and $S_\Sigma = -22.3$ MeV$\cdot$fm^5 at $k_F = 1.35$ fm^{-1} into various contributions. The model is FSS. The unit is MeV $\cdot$ fm^5.

		$I = 1/2$		$I = 3/2$	
		odd	even	odd	even
S_Λ	LS	-17.1	0.6	—	—
	$LS^{(-)}$	12.7	0.3	—	—
S_Σ	LS	2.7	0.1	-11.9	-1.6
	$LS^{(-)}$	-10.5	-0.6	0.1	-0.0

and the $LS^{(-)}$ components in even and odd states as well as $I = 1/2$ and $I = 3/2$ channels is shown in Table 8 for $k_F = 1.35$ fm^{-1}. It is clear that in the case of S_Λ the $LS^{(-)}$ contribution almost cancels the LS one just as in the Born approximation, which makes the ratio S_Λ/S_N to be less than 1/10 for $\rho \lesssim \rho_0$. For the Σ hyperon, the contribution from the $LS^{(-)}$ force has an opposite sign to that for the Λ hyperon, and the ratio S_Σ/S_N turns out to be about 1/2 even in the realistic calculation, which is very much independent of the precise value of k_F.

4 Summary

Since the spin-orbit force is the simplest momentum-dependent short-range force in the baryon-baryon interaction, it is sometimes discussed that the quark substructure of baryons might play an essential role as the microscopic origin of this very important non-central force [11,13,18]. In the hyperon-nucleon (YN) interaction, the spin-orbit force has very rich contents, consisting of three different types; LS, $LS^{(-)}$, and $LS^{(-)}\sigma$ [23]. These LS forces predicted by the color-analogue of the Fermi-Breit (FB) interaction in the $(3q)$-$(3q)$ resonating-group method have correct spin and flavor dependence, which is very similar to that predicted by traditional meson-exchange models [12]. As to the magnitude of these LS forces, we have pointed out [11] that the inclusion of the Galilean non-invariant aLS term of the FB interaction is important, since it gives almost one-third of the Galilean invariant sLS term with the same sign. The choice of the harmonic oscillator constant b is also crucial to obtain enough strength of the LS forces. In order to confirm that these LS forces are consistently described with the short-range repulsion, we have proposed several unified models of the NN and YN interactions [3–7], in which a realistic description of these interactions is achieved not only for the LS forces but also for many other components of the central and non-central forces. In these models, the short-range interaction composed of the strongly repulsive central

force and the LS forces is mainly described by the quark-exchange kernel of the FB interaction, and the medium- and long-range interaction composed of the attractive central force and the long-range tensor force is described by meson-exchange processes acting between quarks.

In this paper we have developed a formulation of the single-particle (s.p.) spin-orbit (ℓs) potentials for the nucleon and hyperons, following the idea presented by Scheerbaum [16]. The quark-exchange kernel from the color-analogue of the FB interaction is directly employed to calculate the strength factor S_B for the s.p. ℓs potentials in the Born approximation. In the simplest treatment, S_B is concisely expressed in terms of quark parameters, among which the parameter $\lambda = (m_s/m_{ud})$, representing the flavor-SU_3 symmetry breaking (FSB) at the quark level, plays an important role. Such expressions are very useful for examining the characteristic structure of the s.p. ℓs potentials. The ratio of S_B to the nucleon strength S_N for the spin-saturated $Z = N$ nuclei is found to be $S_\Lambda/S_N \sim 1/5$, $S_\Sigma/S_N \sim 1/2$ and $S_\Xi/S_N \sim -1/4$ in the Born approximation with the full FSB, irrespective of various versions of our quark model. This result is consistent with the estimation by Morimatsu *et al.* [13], $U_N : U_\Lambda : U_\Sigma = 1 : 0.21 : 0.55$, although they used only the Galilean invariant sLS term in the FB interaction. This ratio is also preserved by the Galilean non-invariant aLS term in the FB interaction, but the inclusion of this term makes the magnitude of S_B reasonable for the realistic description, in contrast to the large value presented in [13]. It is interesting to note that Dover and Gal [27] also predicted $V_{SO}^\Lambda/V_{SO}^N \sim 0.2$ and $V_{SO}^\Sigma/V_{SO}^N \sim 0.6$, by using the coupling constants of the Nijmegen model F potential.

We have also developed a formulation to evaluate the S_B factor from the G-matrix solution of our quark-model potential. Here we first calculated NN, ΛN and ΣN G-matrices in symmetric nuclear matter by solving the Bethe-Goldstone equation for the exchange kernel of our quark model FSS [5,6]. These G-matrices are then used to calculate S_B for spin-saturated symmetric nuclear matter, in the same way as the calculation of the s.p. potentials. In the limit of the zero-momentum hyperons, we have found a fairly large reduction of S_Λ, resulting in the ratio $S_\Lambda/S_N \sim 1/12$. For S_N and S_Σ, the effect produced by solving the G-matrix equation is comparatively weak against usual phenomenological potentials with a short-range repulsive core. In particular, we have found $S_N \sim -40$ MeV $\cdot$ fm^5 both in the Born approximation and in the G-matrix calculation. This implies that the effect of the shot-range correlation is rather moderate in the quark-model description of the short-range repulsion.

In the hyperon s.p. ℓs potentials, the antisymmetric LS ($LS^{(-)}$) force originating from the FB spin-orbit interaction (both from the sLS and aLS pieces) plays a characteristic role. If we neglect the FSB, the S_Λ/S_N ratio is already around 1/3. This is because the half of the LS contribution is cancelled by

the $LS^{(-)}$ contribution. The ratio is further reduced to 1/5 by the FSB, originating from the strange to up-down quark-mass difference and the reduction factor of the LS operator due to the difference of N and Λ baryon masses. The former feature of the FSB at the quark level is a special situation of the Λ hyperon, which results from the structure of its spin-flavor SU_6 wave function. Finally, the short-range correlation by solving the G-matrix equation further reduces the ratio to less than 1/10. It may be argued that the ℓs potential is relevant at the surface region in finite nuclei, where the nucleon density is rather low. We have checked that a small deviation of the Fermi-momentum from the value of ordinary symmetric nuclear matter, $k_F = 1.35$ fm^{-1}, does not change this small ratio. Experimental confirmation of the small s.p. ℓs potentials for the Λ hyperon is highly desirable [15].

A Isospin factors $C^I_\tau(B)$ and spatial integrals $f^{LS}_T(\theta)$

In this appendix we give explicit expressions of the isospin factors $C^I_\tau(B)$ in Eq. (13) and the spatial integrals $f^{LS}_T(\theta)$ in Eq. (9). The explicit values of $C^I_\tau(B)$ are

$$\begin{aligned}
& C^I_p(p) = C^I_n(n) = C^I_p(\Xi^0) = C^I_n(\Xi^-) = \delta_{I,1} \quad , \\
& C^I_p(n) = C^I_n(p) = C^I_p(\Xi^-) = C^I_n(\Xi^0) = \frac{1}{2} \qquad \text{for both } I=0 \text{ and } I=1 \quad , \\
& C^{\frac{1}{2}}_p(\Lambda) = C^{\frac{1}{2}}_n(\Lambda) = 1 \quad , \\
& C^I_p(\Sigma^+) = C^I_n(\Sigma^-) = \delta_{I,\frac{3}{2}} \quad , \\
& C^I_p(\Sigma^-) = C^I_n(\Sigma^+) = \delta_{I,\frac{1}{2}}\ \frac{2}{3} + \delta_{I,\frac{3}{2}}\ \frac{1}{3} \quad , \\
& C^I_p(\Sigma^0) = C^I_n(\Sigma^0) = \delta_{I,\frac{1}{2}}\ \frac{1}{3} + \delta_{I,\frac{3}{2}}\ \frac{2}{3} \quad ,
\end{aligned} \tag{A.1}$$

and a sum rule

$$\sum_\tau C^I_\tau(B) = \frac{2I+1}{2I_B+1} \tag{A.2}$$

is satisfied for each B. The spatial functions $f^{LS}_T(\theta)$ are given by [12]

$$f^{LS}_T(\theta) = (-2\pi)\, \alpha_S x^3 m_{ud} c^2\ b^5$$

[12] See Appendix B of [24].

$$
\times \begin{cases} \left(\frac{12}{11}\right)^{\frac{3}{2}} \exp\left\{-\frac{2}{11}b^2\left[\frac{4}{3}(\boldsymbol{q}^2+\boldsymbol{k}^2)-\boldsymbol{k}\cdot\boldsymbol{q}\right]\right\}\ \widetilde{h}_1\left(\frac{1}{\sqrt{11}}b|\boldsymbol{q}+\boldsymbol{k}|\right) \\ \left(\frac{12}{11}\right)^{\frac{3}{2}} \exp\left\{-\frac{2}{11}b^2\left[\frac{4}{3}(\boldsymbol{q}^2+\boldsymbol{k}^2)+\boldsymbol{k}\cdot\boldsymbol{q}\right]\right\}\ \widetilde{h}_1\left(\frac{1}{\sqrt{11}}b|\boldsymbol{q}-\boldsymbol{k}|\right) \\ \left(\frac{3}{4}\right)^{\frac{3}{2}} \exp\left\{-\frac{1}{3}b^2\left(\boldsymbol{q}^2+\frac{1}{4}\boldsymbol{k}^2\right)\right\}\ \widetilde{h}_1\left(\frac{1}{2}b|\boldsymbol{k}|\right) \\ \exp\left\{-\frac{1}{3}b^2\boldsymbol{k}^2\right\}\ \widetilde{h}_1\left(\frac{1}{\sqrt{3}}b|\boldsymbol{q}|\right) \end{cases}
$$

$$
\text{for}\quad \mathcal{T} = \begin{cases} S \\ S' \\ D_+ \\ D_- \end{cases}, \tag{A.3}
$$

where $\boldsymbol{k} = \boldsymbol{q}_f - \boldsymbol{q}_i$, $\boldsymbol{q} = (1/2)(\boldsymbol{q}_f + \boldsymbol{q}_i)$, $\cos\theta = \widehat{\boldsymbol{q}}_f \cdot \widehat{\boldsymbol{q}}_i$, and $\widetilde{h}_1(x)$ is defined as

$$
\widetilde{h}_1(x) = 3e^{-x^2}\int_0^1 e^{x^2t^2}t^2dt = 1 + 3\sum_{n=1}^{\infty}(-1)^n\frac{(2x^2)^n}{(2n+3)!!}\quad . \tag{A.4}
$$

Here $\widetilde{h}_1(x)$ is normalized as $\widetilde{h}_1(0) = 1$. If we set $\boldsymbol{q} = 0$ in Eq. (A.3), it is simplified to

$$
\overline{f_{\mathcal{T}}^{LS}(\theta)} \sim f_{\mathcal{T}}^{LS}(\theta)|_{\boldsymbol{q}=0} = (-2\pi)\,\alpha_S x^3 m_{ud}c^2\ b^5
$$

$$
\times \begin{cases} \left(\frac{12}{11}\right)^{\frac{3}{2}} \exp\left\{-\frac{8}{33}(bk)^2\right\}\ \widetilde{h}_1\left(\frac{1}{\sqrt{11}}bk\right) \\ \left(\frac{3}{4}\right)^{\frac{3}{2}} \exp\left\{-\frac{1}{12}(bk)^2\right\}\ \widetilde{h}_1\left(\frac{1}{2}bk\right) \\ \exp\left\{-\frac{1}{3}(bk)^2\right\} \end{cases} \quad \text{for}\quad \mathcal{T} = \begin{cases} S,\ S' \\ D_+ \\ D_- \end{cases}, \tag{A.5}
$$

where we assume $k = \overline{q} \sim (2\pi/4t) \sim 0.7\ \text{fm}^{-1}$. The analytic expression of S_B in Eq. (52) is easily derived, if we further set $\overline{q} = 0$.

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